

Non-Linear Optimization of Critical Path Method

Yutika Amelia Effendi, Riyanarto Sarno

Department of Informatics

Faculty of Information Technology, Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia

e-mail: yutika.effendi@gmail.com, riyanarto@if.its.ac.id

Abstract—Time and Cost Optimization (TCO) may be described as a process to identify appropriate construction activities to get the minimum duration of process and minimum additional cost which are needed. Critical Path Method (CPM) crashing project is one of methods to solve the time and cost optimization. But CPM crashing project needs some data to speed up the process business and in reality there are a lot of data which are represented using double timestamp event log. Therefore this paper represents a method to get data which are used in the crashing project. The data are obtained from average execution of activity duration in each case of event log. Then, to do the crashing project, this paper uses non-linear modeling to get minimum activity duration and minimum additional cost.

Keywords—time and cost optimization; CPM crashing project; double timestamp event log; non-linear modeling

I. INTRODUCTION

Cost effective in business process is an important issue in optimization problem [1]. There are two basis important domains in cost effective, namely management of project time and management of project cost [2]. Problem of Time and Cost Optimization (TCO) is a problem which describes crossover of the two basis important domains [3].

In the late 1950s, Critical Path Method (CPM) was introduced. Researchers and practitioners give attention to the TCO and they have produced many publications including the areas of optimization models and optimization methods. CPM crashing project is one of the methods to solve time and cost tradeoff problem. Time and cost optimization problem itself has two mainly aspects; the activity duration which needs to be decreased [4] and amount of activity duration which needs to be decreased [5].

To solve the TCO problem using CPM, many different ways of optimization methods have been proposed. The model of CPM is represented using either exact mathematical programming methods or approximate heuristics methods. This paper uses non-linear modeling to make model of CPM crashing project. There are a lot of model of data which represents business process. Double timestamp event log is one of the data which often exists in real world [6]. Double timestamp event log has the data of activity which exists in business process, execution of activity duration in each case of event log; start time and end time, and the activity cost in each case of event log [7], but double timestamp event log does not

represent data which can used to do the crashing project, therefore this paper represents a method to calculate data from double timestamp event log which is used to do the crashing project.

II. RELATED WORK

A. Business Process Model

A set of linked activities which is produced for specific service is the definition of the business process [8]. Business process has information about where and when the activities are executed, input and output of activity, initial condition before activity is executed and final condition after activity is executed [9]. The characteristics of the business process itself are as follows:

- 1) Have a specific purpose
- 2) Have a specific input
- 3) Have a specific output
- 4) Utilize resource
- 5) Have an activity that can be executed in a certain order [12]
- 6) It can involve more than one organization

Business process model can represents business process correctly [10]. There are a lot of ways to represent business process model, such as UML, Causal Net, BPEL, BPMN, EPC, PNML, etc [14]. Each type of model has different characteristic, such as Petri Net uses token to connect the activity in the business process model, while in Causal Net, the activity can be connected directly.

B. Critical Path Method (CPM)

One of algorithms for process planning is Critical Path Method (CPM). Critical path is obtained from the longest path from a process model. CPM is needed to discover critical path of a process model. The transformation of critical path gives big impact to total duration of the process model. If an activity is late, then the whole process may be late too. In contrast, if an activity is faster than usual, then the whole process is faster than usual too. Therefore, the information about critical path is really important. CPM is frequently used in business process optimization [11].

One of CPM's techniques is crash duration. Crash duration is a technique to crash activity duration and activity cost using linear modeling. Crash duration is used to get the –

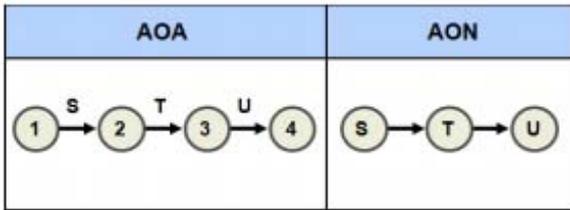


Fig. 1. Serial Form.

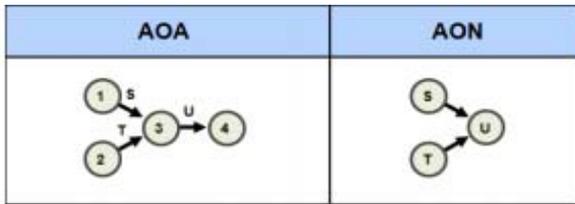


Fig. 2. Join Form.

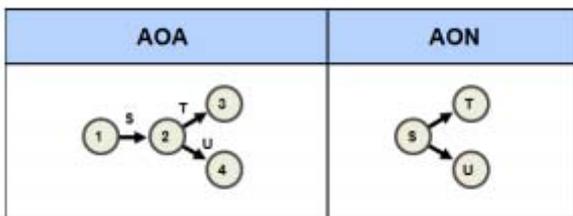


Fig. 3. Split Form.

smallest duration of business process. Resource allocation from a non-critical path to the critical path can fasten the duration of business process. However, this action affects the quality of produced product.

Critical path diagram is obtained from a network (network diagram) which shows the relationship and sequence of activities in a project. In general, network diagram is drawn using the Activity on Node (AON) and Activity on Arrow (AOA) [11].

In the AON, the project activity is represented by a point (node), while in the AOA, the project of activity is represented by arrows (arrow). Project activities that precede/ a requirement to do any other activity referred to predecessor. Fig. 1, to Fig. 3, show the relationship in the project activity using AOA and AON.

In Fig. 1, (serial form) is shown three project activities, namely S, T, and U. In Fig. 1, is shown that the activity of S is predecessor for T activity, while the activity of T becomes predecessor for activity U. In Fig. 2, (join form), it was shown that the activity of S and T be predecessor for U of activity, or in other words, it can be said that the activity could be carried out if the U, S and T activity has been carried out beforehand. In Fig. 3, (split form) shows activity to activity predecessor S into T and U. This shows that the activity of T and U, S can be implemented if the activity has been carried out beforehand.

This paper uses the type of AON critical path on a network or business processes for determining the duration of the business process. Besides being used to determine the length of one of the business process, CPM crashing project method is also used as the basis for optimization duration. The basis is the calculation of cost slope and time slope, which will be used to be a constant role in the mathematical model of the non-linear modeling [15].

C. Optimization using Linear Programming

From the calculation of optimization data in [16], then they use linear programming to optimize the data to get minimum duration with minimum additional cost. The steps are:

- 1) Calculate the normal duration and the minimum duration of business process.
- 2) Find the critical path from business process using Critical Path Method.
- 3) Choose the activity with the smallest crash cost from the critical path.
- 4) Calculate maximum crash amount of the chosen activity.
- 5) Crash the chosen activity with the maximum crash amount.

$$(t_{ni} - \delta) \leq t_{mi} \text{ and } \sum_{j=0}^k t_{n_i} \geq \sum_{j'=0}^{k'} t_{n_{i'}} \quad (4)$$

- 6) Update normal duration of the chosen activity.
- 7) Update normal duration of business process.
- 8) Update additional cost (z) using linear modelling.

$$\text{Min } z = \sum_{i \in A} a_i \delta_i \quad (5)$$

Constraint:

$$t_{n_i} + \delta_i \geq t_{n_i}$$

$$\delta_i \leq y_i$$

$$t_{n_i} \geq 0$$

- 9) Do step 1-8 until reaching the minimum duration.

III. THE PROPOSED METHOD

There are two main steps in this method. First, calculate optimization data and then, optimize the data using non-linear modeling.

A. Step in Calculate Optimization Data

Business process model optimization which uses CPM crashing project needs two types of time and two types of cost. Types of time and cost which are needed in crashing project are:

- Normal Duration
The time which is required to complete an activity with normal existing resources without additional cost in a project.
- Crash Duration

The time which is required to shorten the duration is shorter than the normal duration.

- Normal Cost
The cost which is incurred by project completion in normal duration.
- Crash Cost
The cost which is incurred by project completion within the period of duration of its crash. The cost after crashing project would be greater than the normal cost.

All activities in business process should have the two types of time and two types of cost. Double timestamp event log does not know the information about the normal duration, crash duration, and crash cost of each activity. Therefore this paper proposes a method to calculate optimization data using non-linear modeling.

a. Calculate Normal Duration and Normal Cost

The normal activity duration is obtained from average the activity duration in each case of event log. Meanwhile, the normal activity cost is obtained from average the activity cost in each case of event log. Before we calculate the normal activity duration, we need to calculate the execution of activity duration which is obtained by subtracting the activity duration 1 with the activity duration 2. For example, activity Y is output of activity X, then the execution of activity duration X is subtraction of activity duration Y with activity duration X.

$$\text{if activity}_{\text{output}} = \text{tru then ed} = \text{et}_{\text{activity}_{\text{output}}} - \text{et}_{\text{activity}} \quad (1)$$

where the symbols mean:

- ed : execution of activity duration
- $\text{et}_{\text{activity}_{\text{output}}}$: activity duration 1
- $\text{et}_{\text{activity}}$: activity duration 2

After we get all execution of activity duration in all cases of event log, then we calculate the normal duration of activity using (2).

$$dn_{act} = \frac{\sum_{i=1}^m ed_i}{m} \quad (2)$$

where the symbols mean:

- dn_{act} : normal activity duration
- ed_i : execution of activity duration in case i
- m : frequency of activity in event log (case that contain the activity)

After we calculate the normal activity duration, then we calculate the normal activity cost. Event log contains the activity cost in each case of event log. Therefore, to get the normal activity cost, we calculate average the activity cost in each case using (3).

$$cn_{act} = \frac{\sum_{i=1}^m cost_i}{m} \quad (3)$$

where the symbols mean:

- cn_{act} : normal activity cost
- $cost_i$: activity cost in case i

b. Calculate Normal Cost and Crash Cost

The crash time of each activity is obtained by reducing normal activity duration with standard deviation of execution

of activity duration. And to determine the crash cost of each activity, we add normal cost with standard deviation of the activity cost.

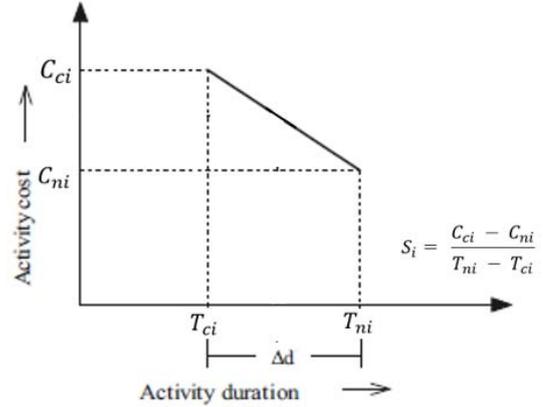


Fig. 4. Cost Slope.

c. Calculate Upper Bound and Lower Bound

We need to know the upper bound and lower bound of the activity duration. The activity duration will be divided properly by using a range of lower bound and upper bound [13]. We can determine upper bound and lower bound using (4).

$$\begin{aligned} \text{lower bound} &\leq \text{activity duration}(i,j) \leq \\ \text{upper bound} &\quad \text{for all } (i,j) \in \text{Activity} \end{aligned} \quad (4)$$

Constraint:

- lower bound : lower bound activity duration (i,j);
- $\text{lower bound} \geq 0, \text{lower bound} \leq \text{upper bound}$
- upper bound : upper bound activity duration (i,j);
- $\text{upper bound} \geq 0$

d. Calculate Time Slope and Cost Slope

Time and cost tradeoff has priority to activities which have the lowest time slope and the lowest cost slope. Time slope is the maximum time which can be generated for the crashing project. To calculate the time slope, we can use (5).

$$X_i = T_{ni} - T_{ci} \quad (5)$$

where the symbols mean:

- X_i : time slope of activity i, $i \in \{\text{activity}\}$
- T_{ni} : normal activity duration i, $i \in \{\text{activity}\}$
- T_{ci} : crash duration of activity i, $i \in \{\text{activity}\}$

Cost slope is the value ratio between all the increased costs. We obtain the cost slope by accelerating project completion time. To calculate the cost slope, we can use (6). In Fig. 4, we show the illustration of cost slope.

$$S_i = \frac{C_{ci} - C_{ni}}{T_{ni} - T_{ci}} \quad (6)$$

where the symbols mean:

- S_i : cost slope of activity i, $i \in \{\text{activity}\}$
- C_{ni} : normal activity cost i, $i \in \{\text{activity}\}$
- C_{ci} : cost crash of activity i, $i \in \{\text{activity}\}$

T_{ni} : normal activity duration i , $i \in \{\text{activity}\}$
 T_{ci} : crash duration of activity i , $i \in \{\text{activity}\}$

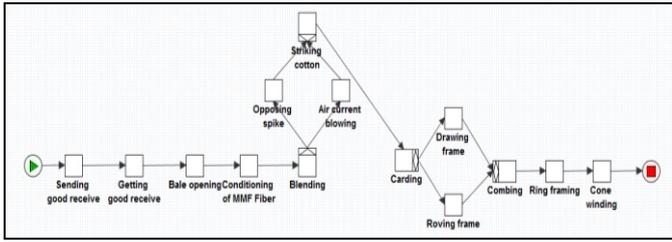


Fig. 5. Process Model.

| Case ID | Activity | Time Stamp | Time Stamp2 | Organisator | Cost |
|---------|---------------------------|------------------|------------------|---------------------|--------|
| PP1 | Sending good receive | 20/06/2014 08.32 | 20/06/2014 13.42 | Purchase Department | 243303 |
| PP1 | Getting good receive | 20/06/2014 13.42 | 20/06/2014 23.41 | Spinning Department | 138941 |
| PP1 | Bale opening | 20/06/2014 23.41 | 21/06/2014 08.16 | Spinning Department | 508599 |
| PP1 | Conditioning of MMP Fiber | 21/06/2014 08.16 | 21/06/2014 10.46 | Spinning Department | 637006 |
| PP1 | Blending | 21/06/2014 10.46 | 21/06/2014 16.57 | Spinning Department | 496301 |
| PP1 | Oposing spike | 21/06/2014 16.57 | 21/06/2014 18.09 | Blowing Department | 352814 |
| PP1 | Striking cotton | 21/06/2014 18.09 | 21/06/2014 19.15 | Blowing Department | 125149 |
| PP1 | Carding | 21/06/2014 19.15 | 22/06/2014 10.51 | Spinning Department | 957335 |
| PP1 | Roving frame | 22/06/2014 10.51 | 22/06/2014 16.28 | Framing Department | 516512 |
| PP1 | Drawing frame | 22/06/2014 16.28 | 22/06/2014 23.42 | Framing Department | 455652 |
| PP1 | Combing | 22/06/2014 23.42 | 23/06/2014 04.48 | Spinning Department | 394780 |
| PP1 | Ring framing | 23/06/2014 04.48 | 23/06/2014 16.44 | Spinning Department | 359985 |
| PP1 | Cone winding | 23/06/2014 16.44 | 23/06/2014 23.26 | Spinning Department | 319064 |
| PP2 | Sending good receive | 23/06/2014 23.26 | 24/06/2014 04.19 | Purchase Department | 212893 |
| PP2 | Getting good receive | 24/06/2014 04.19 | 24/06/2014 07.48 | Spinning Department | 157720 |
| PP2 | Bale opening | 24/06/2014 07.48 | 25/06/2014 01.08 | Spinning Department | 405832 |
| PP2 | Conditioning of MMP Fiber | 25/06/2014 01.08 | 25/06/2014 03.02 | Spinning Department | 137116 |
| PP2 | Blending | 25/06/2014 03.02 | 25/06/2014 05.11 | Spinning Department | 450040 |
| PP2 | Air current blowing | 25/06/2014 05.11 | 25/06/2014 08.25 | Blowing Department | 269791 |
| PP2 | Striking cotton | 25/06/2014 08.25 | 25/06/2014 12.45 | Blowing Department | 186637 |
| PP2 | Carding | 25/06/2014 12.45 | 26/06/2014 00.12 | Spinning Department | 107902 |
| PP2 | Drawing frame | 26/06/2014 00.12 | 26/06/2014 04.48 | Framing Department | 460565 |
| PP2 | Combing | 26/06/2014 04.48 | 26/06/2014 15.16 | Spinning Department | 417705 |
| PP2 | Ring framing | 26/06/2014 15.16 | 26/06/2014 22.49 | Spinning Department | 360005 |
| PP2 | Cone winding | 26/06/2014 22.49 | 27/06/2014 05.19 | Spinning Department | 313302 |

Fig. 6. Fragment of event log.

B. Optimization Using Non-Linear Modeling

From the calculation of optimization data in section III A, then we use non-linear modeling to optimize the data to get minimum duration with minimum additional cost. The steps of non-linear modeling are explained as follows:

- 1) Calculate normal duration and minimum duration of business process.
- 2) Calculate normal cost and crash cost of business process.
- 3) Calculate the time slope and cost slope of business process.
- 4) Calculate the upper bound and lower bound.
- 5) Find critical path from business process using Critical Path Method.
- 6) Calculate the coefficient of time distribution using (7)

$$\text{Coefficient of time distribution} = \frac{\text{normal duration} - \text{crash duration}}{\text{duration}} \quad (7)$$

- 7) Update normal duration of the chosen activity.
- 8) Update normal duration of business process.
- 9) Update additional cost (Min Z) using non-linear modeling using (8).

$$\text{Min } Z = \sum(\text{Cost Slope of Activity } i * (\text{Normal Duration of Activity } i - \text{Crash Duration of Activity } i)^2) \quad (8)$$

IV. EVALUATION

The experiment uses process model in Fig. 5. Process model in Fig. 5, has event log such as in Fig. 6. Event log in Fig. 6, does not has data which are compatible to –

TABLE I. DATA CALCULATION RESULTS USING (1), (2), (3), (6)

| Activity | Normal Duration | Cost Slope | Normal Cost | Crash Cost |
|---------------------------|-----------------|------------|-------------|------------|
| Sending Good Receive | 5.375 | 122637.591 | 338087.1 | 569644.97 |
| Getting Good receive | 6.289 | 125764.061 | 271865.8 | 537229.43 |
| Bale opening | 11.112 | 43418.041 | 384192.4 | 576546.51 |
| Conditioning of MMP Fiber | 2.302 | 443420.542 | 250027.7 | 477389.29 |
| Air current blowing | 3.557 | 341947.435 | 588024 | 923573.86 |
| Striking cotton | 4.601 | 141466.417 | 370134.9 | 612841.43 |
| Carding | 10.668 | 90604.781 | 439227.9 | 693768.57 |
| Drawing frame | 8.797 | 92198.827 | 452087.86 | 683999.75 |
| Combing | 8.698 | 64004.602 | 425811.1 | 609426.25 |
| Ring framing | 9.537 | 43531.483 | 331152.2 | 513673.51 |
| Cone winding | 5.569 | 68066.629 | 365419.4 | 555446.12 |
| Blending | 4.476 | 73385.306 | 346643.3 | 535355.06 |

TABLE II. DATA CALCULATION RESULTS USING (4), (5)

| Activity | Lower Bound | Upper Bound | Time Slope |
|---------------------------|-------------|-------------|------------|
| Sending Good Receive | 2.173 | 8.174 | 1.889 |
| Getting Good receive | 3.488 | 9.989 | 2.112 |
| Bale opening | 5.247 | 18.561 | 4.431 |
| Conditioning of MMP Fiber | 1.639 | 3.029 | 0.513 |
| Air current blowing | 2.805 | 5.166 | 0.981 |
| Striking cotton | 1.108 | 6.714 | 1.716 |
| Carding | 6.772 | 15.591 | 2.809 |
| Drawing frame | 4.602 | 12.175 | 2.515 |
| Combing | 5.097 | 13.397 | 2.869 |
| Ring framing | 2.587 | 15.206 | 4.193 |
| Cone winding | 0.651 | 7.369 | 2.792 |
| Blending | 0.584 | 8.096 | 2.572 |

use in optimization using CPM crashing project. Therefore, we use (1), (2), (3), and (6) to get the optimization data.

The data which are generated using (1), (2), (3), and (6) are shown in Table I. After we calculate the normal duration, cost slope, normal cost, and crash cost, then we calculate the upper

bound, lower bound, and time slope using (4) and (5). The results are shown in Table II.

From the calculation results, we get the critical path from the process model:

Sending good receive -> Getting good receive -> Bale opening -> Conditioning of MMP Fiber -> Air current blowing -> Striking cotton -> Carding -> Drawing frame -> Combing -> Ring framing -> Cone winding -> Blending.

After using time prediction to the process model, we get the critical path and the process model is ready to be discovered using the proposed optimization method. The process model is shown on Fig. 7.

We get the critical path, then we calculate the coefficient of time distribution using (7).

- Sum Coefficient = 20.00001 (where the reduced the time that we want)

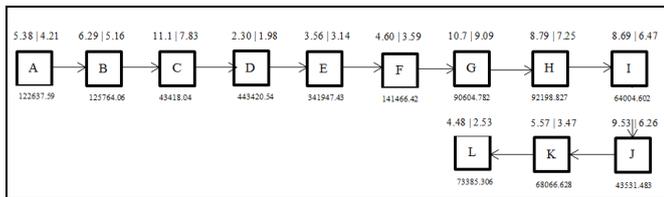
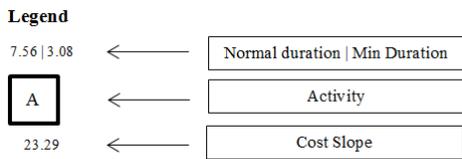


Fig. 7. Process Model after optimization.

The labels of activity in process model after optimization mean:

- A : sending good receive
- B : getting good receive
- C : Bale opening
- D : conditioning of MMP Fiber
- E : air current blowing
- F : striking cotton
- G : carding
- H : drawing frame
- I : combing
- J : ring framing
- K : cone winding
- L : blending

Coefficient will be squared (using the objective function of the calculation). Small coefficient for time slope large activities and large coefficient for time slope small activities. It is intended to get the minimum Z value with non-linear modeling. The results are shown in Table III.

Using (8), we calculate the minimum additional cost using non-linear modeling each activity in process model. Minimum additional cost is calculated for each activity which belong to the critical path in process model.

- Min additional Cost Conditioning of MMP Fiber
 = Cost Slope Conditioning of MMP Fiber * (Coefficient of Conditioning of MMP Fiber)²
 = 443420.542 * (0.322)²
 = 45904.538
- Min additional Cost Air current blowing

$$= \text{Cost Slope Air current blowing} * (\text{Coefficient of Air current blowing})^2$$

$$= 341947.435 * (0.417)^2$$

$$= 59526.508$$

- Min additional Cost Striking cotton
 = Cost Slope Striking cotton * (Coefficient of Striking cotton)²
 = 141466.417 * (1.009)²
 = 143885.561

TABLE III. COEFFICIENT OF CALCULATION RESULTS

| Activity | Normal Duration | Crash Duration | Coefficient of Time Distribution |
|---------------------------|-----------------|----------------|----------------------------------|
| Sending Good Receive | 5.375 | 4.212 | 1.163 |
| Getting Good receive | 6.289 | 5.155 | 1.134 |
| Bale opening | 11.112 | 7.826 | 3.286 |
| Conditioning of MMP Fiber | 2.302 | 1.979 | 0.322 |
| Air current blowing | 3.557 | 3.139 | 0.417 |
| Striking cotton | 4.601 | 3.592 | 1.009 |
| Carding | 10.668 | 9.093 | 1.575 |
| Drawing frame | 8.797 | 7.249 | 1.547 |
| Combing | 8.698 | 6.468 | 2.229 |
| Ring framing | 9.537 | 6.259 | 3.277 |
| Cone winding | 5.569 | 3.474 | 2.096 |
| Blending | 4.476 | 2.532 | 1.944 |

- Min additional Cost Getting Good Receive
 = Cost Slope Getting Good Receive * (Coefficient of Getting Good Receive)²
 = 125764.061 * (1.134)²
 = 161850.293
- Min additional Cost Sending Good receive
 = Cost Slope Sending Good receive * (Coefficient of Sending Good receive)²
 = 122637.59 * (1.163)²
 = 165976.513
- Min additional Cost Drawing frame
 = Cost Slope Drawing frame * (Coefficient of Drawing frame)²
 = 92198.827 * (1.547)²
 = 220772.317
- Min additional Cost Carding
 = Cost Slope Carding * (Coefficient of Carding)²
 = 90604.781 * (1.575)²
 = 224656.605
- Min additional Cost Blending
 = Cost Slope Blending * (Coefficient of Blending)²
 = 73385.306 * (1.944)²
 = 277370.999
- Min additional Cost Cone winding
 = Cost Slope Cone winding * (Coefficient of Cone winding)²
 = 68066.629 * (2.096)²
 = 299044.529
- Min additional Cost Combing

$$= \text{Cost Slope Combing} * (\text{Coefficient of Combing})^2$$

$$= 64004.602 * (2.229)^2$$

$$= 318023.348$$

- Min additional Cost Ring framing
 - = Cost Slope Ring framing * (Coefficient of Ring framing)²
 - = 43531.483 * (3.277)²
 - = 467591.491

TABLE IV. PROCESS MODEL OPTIMIZATION RESULTS

| Activity | Cost Slope | Coefficient of Time Distribution | Additional Cost |
|---------------------------|------------|----------------------------------|-----------------|
| Conditioning of MMP Fiber | 443420.543 | 0.322 | 45904.538 |
| Air current blowing | 341947.435 | 0.417 | 59526.508 |
| Striking cotton | 141466.417 | 1.009 | 143885.561 |
| Getting Good Receive | 125764.061 | 1.134 | 161850.293 |
| Sending Good receive | 122637.59 | 1.163 | 165976.513 |
| Drawing frame | 92198.827 | 1.547 | 220772.317 |
| Carding | 90604.781 | 1.575 | 224656.605 |
| Blending | 73385.306 | 1.944 | 277370.999 |
| Cone winding | 68066.629 | 2.096 | 299044.529 |
| Combing | 64004.602 | 2.229 | 318023.348 |
| Ring framing | 43531.483 | 3.277 | 467591.491 |
| Bale opening | 43418.041 | 3.286 | 468813.436 |

- Min additional Cost Bale opening
 - = Cost Slope Bale opening * (Coefficient of Bale opening)²
 - = 43418.041 * (3.286)²
 - = 468813.436

After we get the minimum additional cost of each activity, then normal duration of process model reaches its minimum duration. Therefore, optimization process meets the stopping criteria. The optimization process of process model on Fig. 5 is shown in Table IV. Total minimum additional cost is 2853416.138. Finally, to optimize the time becomes 71.83 (hours), it takes additional cost as much as Rp 2,853,415.853 (most optimal).

If we compare the results using linear modelling as proposed in [16], the optimized time is 87.11 (hours) and it takes additional cost as much as Rp 2,751,666.4804 (most optimal). In this case, using non linear modelling we can speed up the time 15.28 (hours) and it costs more Rp 101.749,3726 to finish all the activity in business process.

V. CONCLUSION

This paper presents the method to calculate optimization data in double timestamp event log in business process model. Data which can be calculated using this method is normal activity duration, crash duration, normal activity cost, and crash cost. The normal duration and normal cost are obtained

from average activity duration in each case of event log. Meanwhile, crash duration and crash cost are obtained by reducing and adding normal duration and normal cost with standard deviation. After we get all data, then non-linear modeling are used to get minimum duration and minimum additional cost. From the experiment results, we can conclude that the time becomes 71.83 hours and the cost becomes Rp 2,853,415.853 (most optimal). As future work, we intend to improve the calculation for both time and cost and create a new optimization tool.

ACKNOWLEDGMENT

We would like to thank Department of Informatics, Faculty of Information Technology, Institut Teknologi Sepuluh Nopember for supporting the research.

REFERENCES

- [1] J.W. Zhang Jingwen, Y. Xu, and Z.W. He, etc, "A Review on the time/cost trade-Offs problem in project scheduling," *Journal of Industrial Engineering and Engineering Management*, vol. 1, no. 21, pp. 92-97, 2007.
- [2] J.W. Zhang Jingwen, Y. Xu, and Z.W. He, "Discrete time/cost trade-offs in project scheduling with time-switch constraints," *Chinese Journal of Management Science*, vol. 2, no. 14, pp. 58-64, 2006.
- [3] X.L. Wu, and Z. Yin, "Solving time-cost trade-off model for activity network by minimum cost flow principle," *Journal of Huazhong University of Science and Technology(Nature Science Edition)*, vol. 1, no. 35, pp. 42-45, 2007.
- [4] S.E. Elmaghraby, "On criticality and sentivity in activity networks," *European Journal of Operational Research*, vol. 127, pp. 220-238, 2000.
- [5] J.E. Kelley and M.R. Walker, "Critical path planning and scheduling," In: *Proceedings of the Eastern Joint Computational Conference*, vol. 16, pp. 160-172, 1959.
- [6] R. Sarno, W.A. Wibowo, Kartini, F. Haryadita, Y. Effendi, and K. Sungkono, "Determining model using non-linear heuristics miner and control-flow pattern," *TELKOMNIKA (Telecommunication Computing Electronics and Control)*, vol. 14 (1), 2016. <http://doi.org/10.12928/telkomnika.v14i1.3257>
- [7] R. Sarno, Y. Effendi, and F. Haryadita, "Modified time-based heuristics miner for parallel business processes," *International Review on Computers and Software (IRECOS)*, vol. 11 (3), pp. 249-260, March 2016. <http://doi.org/10.15866/irecos.v11i3.8717>
- [8] Hermawan, and R. Sarno, *Developing Distributed System with Service Resource Oriented Architecture*, TELKOMNIKA (Telecommunication Computing Electronics and Control), 10 (2), pp. 389, 2012. <http://dx.doi.org/10.12928/telkomnika.v10i2.815>
- [9] R. Sarno, and K. Sungkono, "Hidden markov model for process mining of parallel business processes," *International Review on Computers and Software (IRECOS)*, 11 (4), pp. 290-300. 2016. <http://doi.org/10.15866/irecos.v11i4.8700>
- [10] R. Sarno, E.W. Pamungkas, D. Sunaryono, and Sarwosri, "Business process composition based on meta models," *International Seminar on Intelligent Technology and Its Applications (ISITIA)*, 2015. <http://doi.org/10.1109/isitia.2015.7219998>
- [11] W.E. Larson, and F.C. Gray. In *Project Management The Managerial Process Fifth Edition*, pp. 171-180. Oregon State University: McGraw-Hill Irwin, 2006.
- [12] M. Song, and W.M.P. Van Der Aalst, "Towards comprehensive support for organizational mining," *Decision Support Systems*, vol. 46, no. 1, pp. 300-317, 2008.
- [13] W.M.P. van der Aalst. *Process Mining - Discovery, Conformance and Enhancement of Business Processes*, Netherlands: Springer, pp. 95-125, 2011.

- [14] S. Huda, R. Sarno, T. Ahmad, and H.A. Santoso. Identification of Process-based Fraud Patterns in Credit Application. 2nd International Conference on Information and Communication Technology (ICoICT). Pages: 84 – 89, 2014. DOI: 10.1109/ICoICT.2014.6914045
- [15] R.F. Deckro, "Non-Linear Time/Cost Tradeoff Models in Management," Pergamon, vol. 28, pp. 219 - 229, 1995.
- [16] R. Sarno, F. Haryadita, Kartini, Sarwosri, and A. Solichah, "Business Process Optimization from Single Timestamp Event Log," Computer, Control, Informatics and Its Applications (IC3INA), 2015. <https://doi.org/10.1109/ic3ina.2015.7377745>