Petri Net Arithmetic Models for Scalable Business Processes

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***Abstract*—Scalability of business process models can be defined as the growth rate among two business process models. A metric of scalability measurements shows whether the business process models compared are scalable or not. This paper proposed a metric of scalability measure based on similarity metric and complexity measure between two business process models. Petri net is used for modeling the business process. Similarity metric is measured by behavioral and structural petri net models, and also using control flow complexity and cyclomatic complexity for measuring complexity of petri net. Petri net arithmetic models also proposed for modeling petri net as arithmetic. According to the experiment, this paper using 4 petri net arithmetic models to measuring scalability metrics. The result of scalability metric shows that all of petri net arithmetic models are scalable to one another with different growth rate. The growth rate determines how large a model can evolve into more complex models.**

Keywords—control flow complexity; cyclomatic complexity; scalability; petri net arithmetic; similarity metric

I. Introduction

Commonly, scalability is an ability of the process or system to handle the growing number of processes or work. For accommodate that growing number, they have the potential process to be enlarged [1]. Tsai [2] proposed scalability metric for testing the scalability of software as a service (SaaS) applications. On other hands, the scalability metric that shown in [3-5] means that scalability metric has implemented in large areas and an active research topic. This research proposed scalability metric for business process models. For this research, scalability can be defined as metric of growth rate between two business process models. Therefore, metric of scalability measure is important issue for this paper. It can determine two models have been compared is scalable and connectedness or not both.

This paper proposed metric of scalability measure based on similarity metric and complexity measure between two business process models. Petri net is used for modeling the business process. Similarity metric is measured by behavioral and structural petri net models, and also using control flow complexity and cyclomatic complexity for measuring complexity of petri net. Petri net arithmetic models also proposed for modeling petri net as arithmetic. We explain this paper as follow: Section II describes the literature review of this paper, such as petri net arithmetic models, behavioral similarity, structural similarity and scalability of business process. The proposed method of scalability measurement is shown in Section III. Then, Section IV explains about implementation of proposed method with business process cases and closed by conclusion in Section V.

II. Literature Review

A. Petri Net Arithmetic Models

Petri net is business process modeling in order to analyze various models in the business processes [6-8]. A Model of petri net has three elements of model, such as transition, place and arc. The transition usually indicates a particular activity (process step or task) that needs to be fire, or a silent step (i.e., t activity) which is used for routing purposes. Place is an element that used to define the pre-conditions transitions and post-conditions transitions. If the precondition is satisfied, a transition can be fired. The result of each firing from transition will be the post-condition. Transitions and places are always connected by directed arcs in such a way that (i) transitions and places have at least one of directed arc and (ii) for each arc. Transition cannot be connected to another transition and then place cannot be connected to another place [9]. A standard definition for traditional petri nets is explained as below.

A Petri net is a set element (*P, T, F*), where:

- P is element of places.
- T is element of transition, where $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$
- Then, F is element of arch, where $F \subseteq (P \times T) \cup (T \times P)$ denotes a directed arcs that represent a joining places, transitions together, and flow relations.

Fig. 1 and Fig. 2 show the examples of petri net models of sequence model, parallel model and condition model. In sequence model, there is no gateway included. So, flow of activity in sequence model in Fig. 1 is (A-B).

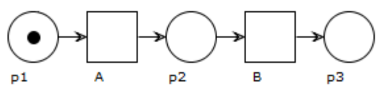

Fig. 1. Examples of Petri Net Sequence Model.

Fig. 2, (a) is petri net parallel model. Parallel model of petri net includes AND-split gateway and AND-join gateway. So, flow of activity should (A-B-C-D). Based on Fig. 2 (b), condition model of petri net includes XOR-split gateway and XOR-join gateway. So, the activity in XOR-split gateway has true value if the flow activity has one choice between two choices of activity. Therefore, flow activity has two condition of processes, (A-B-D) or (A-C-D). Traditional petri net has only XOR gateway, AND gateway without OR gateway include.

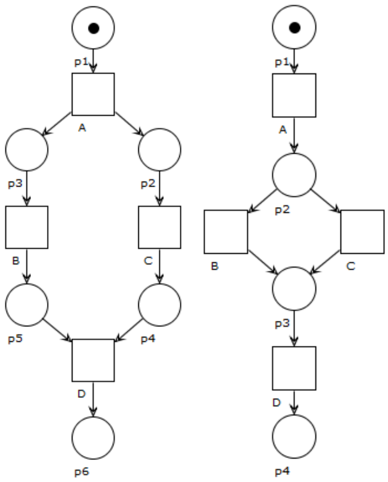

Fig. 2. Examples of Petri Net Parallel (a) and Condition Model (b).

This paper also proposes a new petri net model based arithmetic. It calls petri net arithmetic models. Arithmetic is the well-known and most elementary part of mathematics. Arithmetic terms are used as synonyms for number theory [10]. The addition, multiplication, division and subtraction are the operations of basic arithmetic. This arithmetic also includes more advanced operations, i.e. square roots manipulations, percentages, logarithmic functions, and exponentiation [11].

For petri net model, operators of arithmetic is limited only to a few specific operators. Operator has a function for showing the petri net gateway. A petri net as arithmetic model has two main aspect. There are activity of petri net as operand of arithmetic model and gateway of petri net as operator of arithmetic model. Petri net as an arithmetic model aims as an alternative writing of petri net modeling. Thus, the measurement of similarity between model of petri net and measurement complexity of petri net models can be replaced with arithmetic model that we proposed. Operator arithmetic for petri net gateway is descripted below.

- AND gateway replace with addition (+) operator.
- XOR gateway replace with multiplication (x) operator.
- OR gateway replace with logical disjunction (v) operator.

For example, sequence model of petri net in Fig. 1, can be written as “AB” in arithmetic model without any operator. Parallel model of petri net in Fig. 2 (a) can be written as “A+(BC)+D” in arithmetic model with addition operator for showing the AND gateway. Finally, condition model of petri net in Fig. 2 (b) can be written as “Ax(BC)xD” in arithmetic model with multiplication operator for showing the XOR gateway.

B. Similarity Metric

Similarity metric among two business process models has aims to measure the similarity value by calculating the similarity according to behavioral or structural similarity method. Some of business processes that have a high similarity value should be arranged in such a way to increase efficiency [12]. Structural similarity is one of similarity method to measure similarity based on the elements of business process model. In petri net model, two models will compare according to arcs as the relations between these elements, places and transitions. Other similarity method is measuring similarity value based on the intended behavior of process models. It is called behavior similarity metrics. In petri net, the behavior process models are determined by transitions [13].

Behavioral similarity is measured by compare the transition adjacency relations (TARs) between two models. The equation of behavioral similarity is mentioned in (1).

$$simB = \frac{(Amount\ of\ Similar\ TARset\ )^2}{TARset\ model\ 1\ \ x\ TARset\ model\ 2} \quad (1)$$

For Structure similarity, similarity value is measured by compare the similar aspect of petri net models, such as amount of similar place, similar transitions and similar arcs. The equation of structure similarity is shown in (2):

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} \quad (2)$$

C. Control Flow Complexity (CFC)

Cardoso [14] defines the complexity of process as the complexity degree to analyze, understand or explain process models. In recent, the research communities of software measurement are using complexity as a challenging topic [15]. One of the approaches of complexity measurement in business process models is Control Flow Complexity that proposed by Cardoso [15]. One of the approaches of complexity measurement in business process models is Control Flow Complexity that proposed by Cardoso [14]. Control-Flow Complexity (CFC) can be defined as metric for determining and analyzing the complexity degree of business process. Now, for achieve the effective and efficient process management, analysis of complexity degree for model processes has been an important issue to consider in the quest for it, especially for business process management that using process models. Achieve of effective process management must be the aim of the metric of CFC. CFC metric defines the complexity according to the assumptions which the control-flow behavior of a process is affected by construction of splits and joins. Rolon [16] has implemented CFC metric for BPMN models, Mao [17] has implemented CFC for petri net models, and then Anugrah [18] also has implemented CFC metric for business process decomposition. Therefore, equation is developed to calculate the value of complexity for XOR-split, OR-split, and AND-split. The equation is developed as (3), (4), and (5).

- CFC according to XOR-split is CFC value that constructed the CFC value by the sum of the split out from a process model that defined as the XOR-split. For $CFC_{XOR\text{-}split(m)}$ where m is a process model or an activity that will be measured by the CFC of the XOR-split. For XOR-splits, the CFC is determined by (3):

$$CFC_{XOR-split}(a) = out(a) \quad (3)$$

- CFC according to OR-split is CFC value that constructed by the sum of the split out from a process model that defined as the XOR-split. The CFC for OR-splits is determined by 2^(k-1), where k is the out of the split. It is simply by (4):

$$CFC_{OR-split}(a) = 2^{out(a)-1} \quad (4)$$

- CFC based on AND-split is clear value. The CFC for AND-split is always equal to 1. According to AND-split, it is assumed that all of the out transitions are selected and executed (as the sequence). The equation is simply by (5):

$$CFC_{AND-split}(a) = 1 \quad (5)$$

Therefore, total of CFC metric has been added mathematically. Total of CFC is done by sums up the values of (3), (4), and (5). This is measured as (6).

$$CFC = \sum CFC_{Xor-split}(a) + \sum CFC_{or-split}(a) + \sum CFC_{and-split}(a) \quad (6)$$

D. Cyclomatic Complexity

McCabe cyclomatic complexity (CC) [19] or can be called by McCabe CC is the famous complexity measurement that used widely for metrics of software. CC is corresponding to measuring complexity structure of model process that has been implemented in system of web service [20]. For petri net which modeled by, CC can be measure as (7):

$$CC = |F| - |P| - |T| + 2 \quad (7)$$

The value of CC determines the complexity of control structure in process model [17]. For the running example, its CC value for fig.1 is 4−3 −2+ 2 = 1 because fig 1 has 4 arcs (flow), 3 places and 2 transitions. Normally, sequence type of petri net has value of CC equal to 1.

III. Proposed Scalability Metric

Scalability metric between two model processes is important issue that we discuss in this paper. Scalability metric determines the connectedness and scalable process between two business processes models. If scalability metric between A and B is 0, determine that A and B are not connectedness models and can't scalable. We proposed a equation for measure the scalability metric between two models of business process based on similarity and complexity measure, mentioned in (8):

$$\psi(A,B) = \frac{\sum C(A)}{\sum C(B)} x\ average(simS(A,B), simB(A,B)) \quad (8)$$

Where, measuring S(A) has two alternative equation, by using control flow complexity mentioned in (9) or by using cyclometic complexity mentioned in (10). In section IV, this paper compared both complexities.

$$\sum C(A) = CFC(A)x \sum Structure(A) \quad (9)$$

$$\sum C(A) = CC(A)x \sum Structure(A) \quad (10)$$

- $\psi(A,B)$ is scalability metric between A and B.
- $simS(A,B)$ is structural similarity between A and B.
- $simB(A,B)$ is behavioral similarity between A and B
- $\sum C(A)$ is multiplication of complexity by $CFC(A)$ or $CC(A)$ and $Strucutre(A)$, determine the total complexity.
- $CFC(A)$ states control flow complexity of A, seen from amount of AND, XOR and OR gateway that mentioned in (6).
- $CC(A)$ states cyclometic complexity of A, seen from (7).
- $structure(A)$ states amount of petri net element, such as place, arc and transition.

For measuring scalability metric, this paper uses similarity metric to prove that two models have been compared is connectedness or not and using complexity to measure complexity degree of each model. Based on (8) scalability metric got the conditions that minimum value of scalability metric is equal to 0 and also can't less than 0. It occurred because the similarity metric of structure or behavior has a minimal value of at least 0 and maximum value of 1, this result indicates that two compared models has significant different of business process. It also shows that A and B do not have relationship based on their business processes (not connectedness). Finally, it indicates that A and B are not the scalable business processes. Then, maximum value of scalability metric is equal to 1. It shows similarity value also equal to 1. It indicates that A and B has a maximum similarity, and maximum value of scalability metric.

IV. Discussion

In this section, four models of petri net are shown for testing the scalability metric that has proposed in section III, there are petri net models of A, B, C and D. Model A and Model B are sequential models, but model A has minimum business processes that model B. On other hand, model C is parallel model with AND gateway. Model D is combination between parallel and condition models with XOR gateway and

AND gateway. Scalability metric is determined by measure each model with proposed equation. All of petri net models are shown by Fig. 3 and Fig. 4.

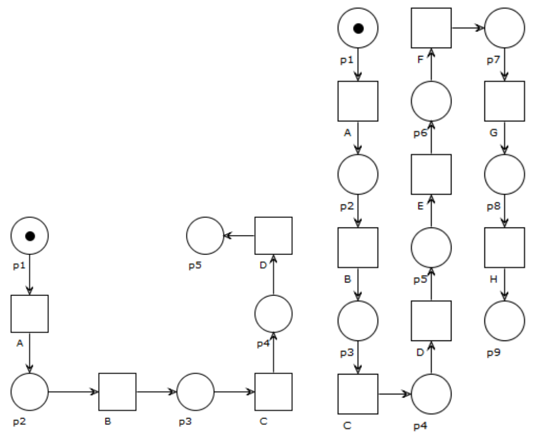

Fig. 3. Petri net model A and petri net model B.

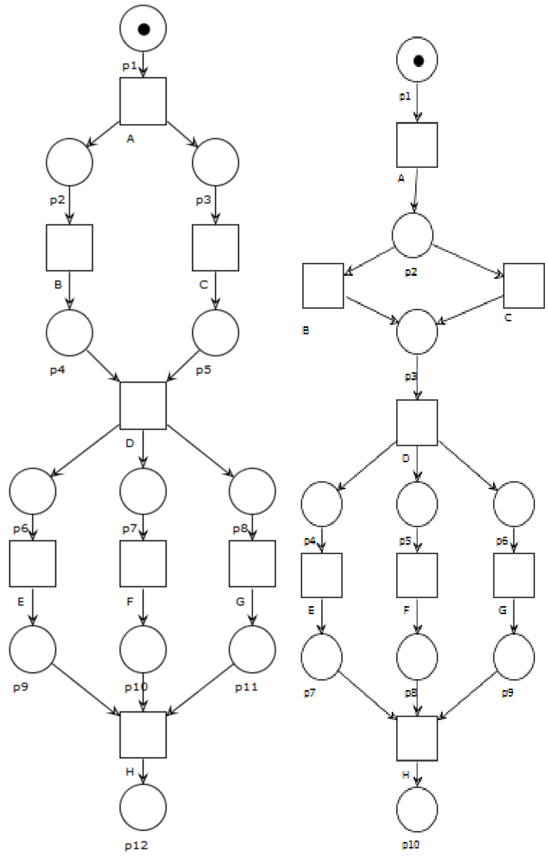

Fig. 4. Petri net model C and Petri net model D.

First, petri net model of A, B, C and D are transformed to petri net arithmetic models. AND gateway replace with addition (+) operator. XOR gateway replace with multiplication (x) operator. Transformed petri nets are shown in Table I.

TABLE I. ARITHMETIC MODELS OF PETRI NETS

Model A	ABCD
Model B	ABCDEFGH
Model C	A+(BC)+D+(EFG)+H
Model D	Ax(BC)xD+(EFG)+H

Then, petri net arithmetic models are measured their similarity by structural similarity and behavioral similarity. Similarity between two business process models has aims to measure the similarity value by calculating the similarity based upon structural and behavioral similarity method. This structural similarity determines metric of similarity by structure of each model, and behavioral similarity determines metric of similarity based on the intended behavior of process models. Value of similarity among two models is in the interval from 0 to 1. If the value is equal to 0, means that two models have been compared has significant different and not connectedness. Vice versa, if the value is equal to 1, indicates that two models have maximum similar. Metric of structural similarity using Jaccard is shown in Table II. Metric of behavioral similarity using TARs is also shown in Table III.

TABLE II. METRIC OF JACCARD STRUCTURAL SIMILARITY

	Model A	**Model B**	**Model C**	**Model D**
Model A	1	0.51	0.31	0.12
Model B	0.51	1	0.41	0.25
Model C	0.31	0.41	1	0.6
Model D	0.12	0.25	0.6	1

TABLE III. METRIC OF TARS BEHAVIORAL SIMILARITY

	Model A	**Model B**	**Model C**	**Model D**
Model A	1	0.43	0.13	0.13
Model B	0.43	1	0.23	0.23
Model C	0.13	0.23	1	1
Model D	0.13	0.23	1	1

Based on the result of similarity metric using jaccard structural or TARs behavioral (Table II and III). All of models have connectedness for one another. It is determined by value of similarity metric, all of them do not have the value equal to 0. Highest connectedness value is owned by sim(A,B), and lowest connectedness by sim(A,D).

Then, determine of total complexity for each model using (9) and (10). Total complexity is measured by multiplying model complexity (CFC or CC) and amount of petri net element structures. Metric of total complexity C using CFC and CC are shown in Table IV and V.

TABLE IV. METRIC OF C USING CFC

Model	**CFC**	$\sum structure$	**C(model)**
C(A)	1	17	17
C(B)	1	33	33
C(C)	1	42	42
C(D)	3	38	114

TABLE V. METRIC OF C USING CC

Model	CC	$\sum structure$	C(model)
C(A)	1	17	17
C(B)	1	33	33
C(C)	4	42	168
C(D)	4	38	152

TABLE VI. METRIC OF SCALABILITY USING CFC

	Model A	Model B	Model C	Model D
Model A	1	0.242121212	0.089048	0.01864
Model B		1	0.251429	0.069474
Model C			1	0.294737
Model D				1

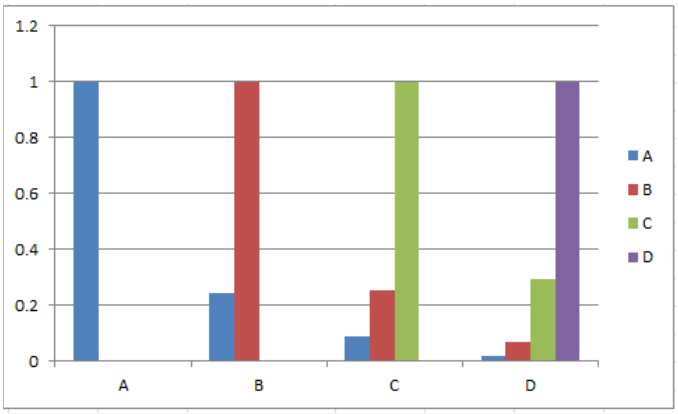

Fig. 5. Chart of scalability metrics using CFC.

TABLE VII. METRIC OF SCALABILITY USING CFC

	Model A	Model B	Model C	Model D
Model A	1	0.242121	0.022262	0.01398
Model B		1	0.062857	0.052105
Model C			1	0.884211
Model D				1

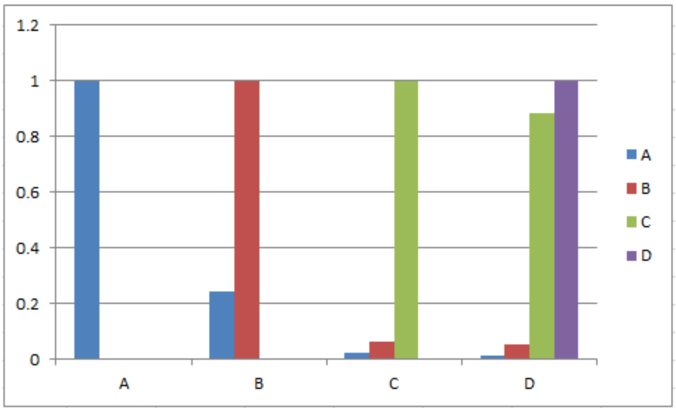

Fig. 6. Chart of scalability metrics using CC.

Based on metric C using CFC and CC (Table IV and V), measurement continues to calculate metric of scalability using (8). Table VI shows value of metric scalability using CFC. And Table VII shows value of metric scalability using CC. Models with low complexity should be compared by models with high complexity to find growth rate between two business process models. Therefore, $\psi(A,C)$ is able to calculate the scalability metric because complexity of A less than complexity of C (see Table IV or Table V). $\psi(A,C)$ calculates the value of scalability metric that indicates model B obtain a growth rate of 0.089 to model A by using CFC and 0.022 by using CC. But, $\psi(C,A)$ is disable to calculate because complexity of C more than complexity of A. Table VI and VII show that all of arithmetic petri net models are scalable for each other. It is determined by result of metric scalability, there are not value equal to 0. Highest scalability metric for different models found on $\psi(C,D)$ by using CFC or CC, then lowest scalability metric for different also found on $\psi(A,D)$ by using CFC or CC. Because $\psi(C,D)$ are scalable models, model C can grow into a model D. Otherwise, the two models can't grow if not scalable models.

V. CONCLUSION AND FUTURE WORK

Scalability metric has proposed to measure the scalability metric between the two models using similarity metric and complexity calculation. Scalability metric determines the two models are scalable or not. In this paper also proposes a new model of petri net based arithmetic. It calls petri net arithmetic models. Based on the experiment using 4 petri net models, scalability metric of each model has measured. All of model are scalable with each other because there aren't scalability metric equal to 0. The value of scalability also indicates the growth rate among two models. If model A and B are scalable models, model A that has lower complexity can grow into a model B. Otherwise, the two models can't grow if not scalable models. The result of scalability metric shows that all of petri net arithmetic models are scalable to one another with different growth rate. Therefore, the growth rate determines how large a model can evolve into more complex models.

Next research, we need to implement the proposed scalability metric to measure the real bases such as measuring the scalability metric of web service that include types of business process. We hope, web service should be developed by determining scalability metric.

ACKNOWLEDGMENT

Authors would like to thank the Department of Informatics, Faculty of Information Technology, Institut Teknologi Sepuluh Nopember Surabaya for supporting this research.

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